AP Calculus BC Notes – Concavity, Points of Inflection, and the Second Derivative Test

Ex. 1 For $f(x)=\frac{1}{4}x^4-6x^2+x-3$, determine:

a. The intervals of concavity.

b. The values of x at which $f(x)$ has a point of inflection.

$f'(x) = x^3 - 12x + 1$

$f''(x) = 3x^2 - 12 = 0$

$x = \pm 2$

f'' + − +

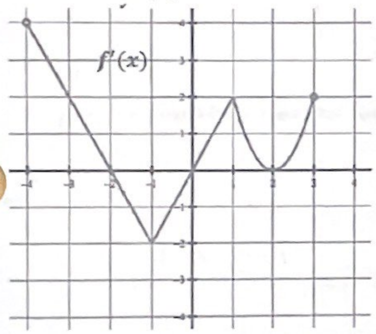
−2 2

f(x) is concave up on $(-\infty,-2)\cup(2,\infty)$ b/c $f''(x) > 0$.

f(x) is concave down on $(-2,2)$ b/c $f''(x) < 0$.

f(x) has a P.o.I. @ $x = -2, 2$ b/c $f''(x)$ changes signs

Ex. 2 Given the graph of $f'(x)$, find the values of x at which the graph of $f(x)$ has a point of inflection.



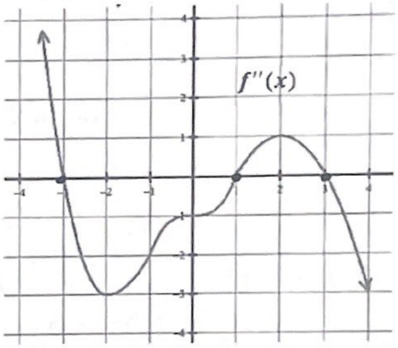
$f'' \rightarrow$ slope of f'

− + − +

−1 1 2

f has P.O.I @ $x = -1, 1, 2$ b/c $f''(x)$ Δs signs.

Ex. 3 Given the graph of $f''(x)$, find the values of x at which the graph of $f(x)$ has a point of inflection.



f'' Δs signs @ $x = -3, 1, 3$.

f has a P.o.I. @ $x = -3, 1, 3$.

Ex. 4 Does the tangent line to the graph of $f(x)=xe^{-x}$ at $x=1$ lie above of below the graph of $f(x)$? Justify your answer.

$f'(x) = -xe^{-x} + e^{-x}$

$= e^{-x}(1-x)$

$f''(x) = -e^{-x} - e^{-x}(1-x)$

$f''(1) = -\frac{1}{e} < 0$

Since $f''(1) < 0$, the tangent line to f(x) @ $x=1$ would lie above the graph of f(x).

The Second Derivative Test for Extrema

a) Indicate the relative extrema on the graph of $f(x)$.

b) What do you know about the value of $f'(x)$ at each extrema?

$f'(x) = 0$

c) What do you know about the value of $f''(x)$ at each extrema?

At max : $f''(x) < 0$

At min : $f''(x) > 0$

Ex. 5 Use the second derivative test to find the relative extrema of $f(x) = x^4 - 2x^2$.

Find Critical Values

$f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$x = 0 \quad x = 1, -1$

$f''(x) = 12x^2 - 4$

Test C.V. in $f''(x)$

$f''(0) = -4 < 0$ → f is concave down

$f''(-1) = 8 > 0$ → f is concave up

$f(1) = 8 > 0$ → f is concave up

$f(x)$ has local min @ $x = -1, 1$ b/c $f'(x) = 0$ & $f''(x) > 0$.

$f(x)$ has local max @ $x = 0$ b/c $f'(0) = 0$ & $f''(0) < 0$

2nd Deriv. Test

Ex. 6 Use the second derivative test to find the relative extrema of $f(x) = \sqrt{2}x - 2\cos x$ on the interval $[0, 2\pi]$.

$f'(x) = \sqrt{2} + 2\sin x = 0$

$\sin x = -\frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$f''(x) = 2\cos x$

$f''(\frac{5\pi}{4}) = -\sqrt{2}$

$f''(\frac{7\pi}{4}) = \sqrt{2}$

f has a local max @ $x = \frac{5\pi}{4}$ b/c $f'(\frac{5\pi}{4}) = 0$ & $f''(\frac{5\pi}{4}) < 0$.

f has a local min @ $x = \frac{7\pi}{4}$ b/c $f'(\frac{7\pi}{4}) = 0$ & $f''(\frac{7\pi}{4}) > 0$.